

14.15. Model: The mass attached to the spring oscillates in simple harmonic motion.

Solve: (a) The period $T = 1/f = 1/2.0 \text{ Hz} = 0.50 \text{ s}$.

(b) The angular frequency $\omega = 2\pi f = 2\pi(2 \text{ Hz}) = 4\pi \text{ rad/s}$.

(c) Using energy conservation

$$\frac{1}{2}kA^2 = \frac{1}{2}kx_0^2 + \frac{1}{2}mv_{0x}^2$$

Using $x_0 = 5.0 \text{ cm}$, $v_{0x} = -30 \text{ cm/s}$ and $k = m\omega^2 = (0.2 \text{ kg})(4\pi \text{ rad/s})^2$, we get $A = 5.54 \text{ cm}$.

(d) To calculate the phase constant ϕ_0 ,

$$A \cos \phi_0 = x_0 = 5.0 \text{ cm}$$

$$\Rightarrow \phi_0 = \cos^{-1}\left(\frac{5.0 \text{ cm}}{5.54 \text{ cm}}\right) = 0.445 \text{ rad}$$

(e) The maximum speed is $v_{\max} = \omega A = (4\pi \text{ rad/s})(5.54 \text{ cm}) = 69.6 \text{ cm/s}$.

(f) The maximum acceleration is

$$a_{\max} = \omega^2 A = \omega(\omega A) = (4\pi \text{ rad/s})(69.6 \text{ cm/s}) = 875 \text{ cm/s}^2$$

(g) The total energy is $E = \frac{1}{2}mv_{\max}^2 = \frac{1}{2}(0.200 \text{ kg})(0.696 \text{ m/s})^2 = 0.0484 \text{ J}$.

(h) The position at $t = 0.40 \text{ s}$ is

$$x_{0.4 \text{ s}} = (5.54 \text{ cm}) \cos[(4\pi \text{ rad/s})(0.40 \text{ s}) + 0.445 \text{ rad}] = +3.81 \text{ cm}$$